PROBLEMS FOR SOLUTION.[1]

SEND ALL COMMUNICATIONS ABOUT PROBLEMS TO B. F. FINKEL, Springfield, Mo.

2660. Proposed by JOSEPH E. ROWE, State College, Pa.

Prove that the distance measured along the side of a triangle, from the point of contact with the inscribed circle to the point of contact with an escribed circle, is equal to the side of the triangle between the two circles.

2661. Proposed by ARTEMAS MARTIN, Washington, D. C.

Find a parallelopipedon whose edges, and the diagonals of its faces, are all rational whole numbers.

2662. Proposed by JOHN LOUKE, New York City.

Assume we have two piles of gold bars. The dimensions of the bars in the first pile are $2.643 \times 5.286 \times 10.573$ and the dimensions of the bars in the second pile are $2.13 \times 6.53 \times 10.573$. If possible, arrange bars from the first pile and from the second pile so as to form perfect cubes, the bars from the piles to be taken separately or in combination.

2663. Proposed by R. P. BAKER, University of Iowa.

From the identity

$$\prod_{k=0}^{\infty}\left(\frac{1}{1-x^{2k+1}}\right) \equiv \prod_{l=1}^{\infty}(1+x^l),$$

Macmahon (*Combinatorial Analysis*, vol. I, p. 10) proves the number of partitions of any integer n into odd parts is equal to the number of partitions of n into parts no two of which are equal.

Taking the classes of some cardinal number, devise an ordinal arrangement which puts each member into one-to-one correspondence with a number of the other class and of such a nature that a direct calculation determines which is the partner of any member in either class.

2664. Proposed by J. W. NICHOLSON, Louisiana State University.

Find the sum of

$$\tfrac{1}{3} - \tfrac{2}{15} + \tfrac{3}{35} - \cdots + (-1)^n \frac{n}{(2n-1)(2n+1)}.$$

2665. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

A telegraph wire, which weighs 1/10 of a pound per yard, is stretched between poles on a level ground, so that the greatest dip of the wire is 3 feet. Find approximately the distance between the poles when the tension at the lowest point of the wire is 140 pounds.

2666. Proposed by W. WOOLSEY JOHNSON, Annapolis, Maryland.

Ten equations between five quantities $x_1, x_2, \cdots, x_5$ being written as follows: $x_1 = 1 - x_3x_4$ and four others formed by cyclic interchange of the suffixes; also $x_5x_1x_2 = x_5 + x_2 - 1$ and four others formed by cyclic interchange; prove that only three of these equations are independent. In other words, the values of x_1 and x_2 being assumed at pleasure, x_3, x_4, and x_5 can be so determined as to satisfy all ten equations.

2667. Proposed by E. L. REES, The University of Kentucky.

Given one diagonal of a parallelogram and the area of the rectangle whose sides are equal to those of the parallelogram, construct the parallelogram so that the diagonal shall make a given angle, α, with a given line and so that the sum of the angles that two adjacent sides make with this line shall be equal to a given angle, β.

[1] Hereafter the problems will be numbered consecutively beginning at the next number above the total number proposed prior to this date. EDITORS.

2668. Proposed by B. F. FINKEL, Drury College.

Show that

$$v = \frac{2}{9}\frac{ga^2\sigma}{\eta},$$

where a is the radius of a droplet, σ its density, η the viscosity of the air and v the velocity under gravity g. Stokes's Law.

2669. Proposed by S. A. COREY, Albia, Iowa.

Let $A_1, A_2, \cdots, A_8$, and $-(A_1 + A_2 + \cdots + A_8)$ be the vector sides of an enneagon, plane or gauche. Also let $B_1, B_2, \cdots, B_8$, and $-(B_1 + B_2 + \cdots + B_8)$ be the vector sides of a second enneagon, where

$$\begin{aligned}
B_1 &= C_1A_1 - C_2C_5A_3 - C_3C_6A_5 + C_4C_5C_6A_7,\\
B_2 &= C_1A_2 - C_2C_5A_4 - C_3C_6A_6 + C_4C_5C_6A_8,\\
B_3 &= C_2A_1 + C_1A_3 - C_4C_6A_5 - C_3C_6A_7,\\
B_4 &= C_2A_2 + C_1A_4 - C_4C_6A_6 - C_3C_6A_8,\\
B_5 &= C_3A_1 + C_4C_5A_3 + C_1A_5 + C_2C_5A_7,\\
B_6 &= C_3A_2 + C_4C_5A_4 + C_1A_6 + C_2C_5A_8,\\
B_7 &= C_4A_1 - C_3A_3 + C_2A_5 - C_1A_7,\\
B_8 &= C_4A_2 - C_3A_4 + C_2A_6 - C_1A_8,
\end{aligned}$$

C_1, C_2, C_3, C_4, C_5, and C_6 being scalars.

Then, if a_s = tensor A_s, b_s = tensor B_s, and $\cos(A_rA_s)$ = cosine of the angle included between A_r and A_s, and $\cos(B_rB_s)$ = cosine of the angle included between B_r and B_s, establish the following relation between the sides and angles of the two enneagons:

$$\begin{aligned}
[C_1^2 + C_5C_2^2 + C_6C_3^2 + C_5C_6C_4^2]\,[a_1a_2\cos(A_1A_2) + C_5a_3a_4\cos(A_3A_4)\\
+ C_6a_5a_6\cos(A_5A_6) + C_5C_6a_7a_8\cos(A_7A_8)]\\
= b_1b_2\cos(B_1B_2) + C_5b_3b_4\cos(B_3B_4) + C_6b_5b_6\cos(B_5B_6) + C_5C_6b_7b_8\cos(B_7B_8).
\end{aligned}$$

Show that Geometry problem 506 is a special case of the foregoing. Give illustrative example, using triangle or other simple geometric figure, by assuming that some of the sides of the first enneagon are zero.

SOLUTIONS OF PROBLEMS.

482 (Algebra). Proposed by C. F. GUMMER, Kingston, Ont.

Find a necessary and sufficient condition that the infinite sequences of positive quantities $(a_1, a_2, \cdots)$ and $(b_1, b_2, \cdots)$ may be such that the series $a_1x_1 + a_2x_2 + \cdots$ and $b_1x_1 + b_2x_2 + \cdots$ either both converge or both diverge, when the x's are any positive quantities.

Solution by the Proposer.

The condition is that $a_1/b_1, a_2/b_2, \cdots$ all lie between two positive limits m and M $(m < M)$. It is sufficient; for it makes $m < (a_rx_r)/(b_rx_r) < M$. It is necessary; for if it does not hold, either the sequence $(a_1/b_1, a_2/b_2, \cdots)$ or $(b_1/a_1, b_2/a_2, \cdots)$ has $+\infty$ for one of its limits. That is (taking the first case), there is a partial sequence $(a_{i_1}/b_{i_1}, a_{i_2}/b_{i_2}, \cdots)$ of increasing quantities tending to ∞. Hence, using an argument due to DuBois-Reymond, the series

$$\sum_{r=2}^{\infty}\left(\sqrt{\frac{b_{i_{r-1}}}{a_{i_{r-1}}}} - \sqrt{\frac{b_{i_r}}{a_{i_r}}}\right)$$

converges, and the series

$$\sum_{r=2}^{\infty}\frac{a_{i_r}}{b_{i_r}}\left(\sqrt{\frac{b_{i_{r-1}}}{a_{i_{r-1}}}} - \sqrt{\frac{b_{i_r}}{a_{i_r}}}\right)$$

diverges. Hence, if

$$x_{i_r} = \frac{1}{b_{i_r}}\left(\sqrt{\frac{b_{i_{r-1}}}{a_{i_{r-1}}}} - \sqrt{\frac{b_{i_r}}{a_{i_r}}}\right) \qquad (r = 2, 3, \cdots),$$